Angular and Linear Speeds

Angular Speed of a Rotating Object

Angular speeds arise where there are rotating objects like car tires, engine parts, computer discs, etc. When you drive a car, a gauge on the dashboard, (called a tachometer), continuously displays how fast your engine, (actually the crankshaft in your engine), is rotating. The number it displays at a particular time is called the angular speed of the engine at that instant, in *revolutions per minute*, (abbreviated to rpm). For example, an angular speed of 3000 rpm means that the engine rotates 3000 times in one minute. Since there isn't much space on the dashboard, 3000 rpm is actually displayed as 3 and you are instructed to multiply it by 1000, (there is symbol $\times 1000$ in the center of the tachometer).

For objects like the seconds hand of a clock, the minute hands of a clock, etc, that rotate relatively slowly, the angular speed is best given in degrees or radians per unit time. For example, the angular speed of the seconds hand of a clock may be given as $360°$ per minute, or $6°$ per second, (divide 3600 by 60), or $21600°$ per hour, (multiply 360 by 60).

In general, to determine the angular speed of a rotating object per unit time do the following: (i) Fix a time interval, (ii) Record the amount of rotation, (in degrees or radians or revolutions), in that time period, (iii) Divide the amount of turning by the length of the time interval.

Exercise 1

1. *What is the angular speed of the hour hand of a clock in:*

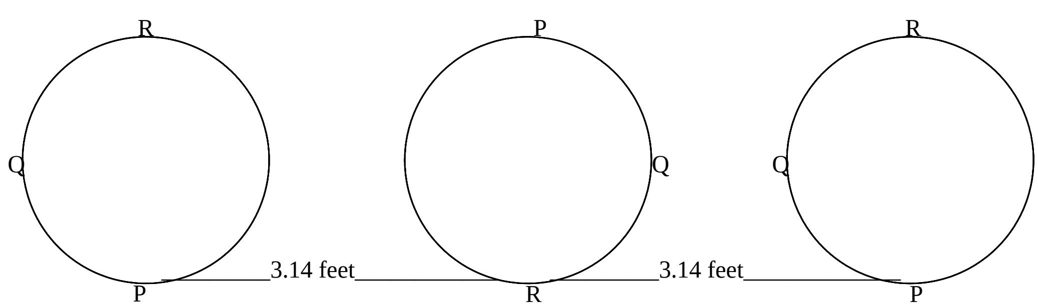
   a. Degrees per hour   b. Degrees per minute   c. Radians per hour   d. Revolutions per minute

2. *Give the angular speed of the earth in degrees per hour and in radians per minute.*

   a. Degrees per hour   b. Degrees per second   c. Radians per minute

Linear Speed of a Rotating Object

The tires of a vehicle must rotate in order for the vehicle to move. When they rotate clockwise, it moves forward. It moves back when they rotate counter-clockwise. Take a typical car with tires that have radius 1 foot each. Imagine the tires making one full rotation clockwise. The figure below shows the position of a tire at the start, then after a half rotation and finally after a full rotation.



At start   After a half rotation   After a full rotation

The car has moved forward $\dfrac{\pi\,(1)\,(180)}{180} = \pi \simeq 3.14$ feet after the first half rotation. It moves another 3.14 feet after the second half rotation. Therefore it moves a total of $2\pi \simeq 6.28$ feet when the tire makes a full rotation. Complete the table

Number of rotations	Distance, in feet, moved by car
1	$2\pi \simeq 6.28$
2	$4\pi \simeq 12.56$
7	
12.5	
	28π
	42π
x	
	150 feet
	750 yards
	1 mile

To get an idea of a linear speed, assume that the tires rotate 6 times every second. Then the vehicle moves $6\,(2\pi) = 12\pi$ feet per second. This is close to 38 feet per second and it is called the *linear speed* of the tires, (and the vehicle) in feet per second. We may transform it into a speed per hour as follows:

Since one hour is equal to 3600 seconds, the tires move $3600 \times 12\pi$ feet in one hour. But 1 mile equals 5280 feet. Therefore the linear speed of the tires is

$$\frac{3600 \times 12\pi}{5280} = 25.7 \text{ miles per hour}$$

In general, the linear speed of a rotating circular object may be calculated as follows: Imagine a tire that has the same radius as the rotating circular object. Fix a time interval t and determine the distance s that the tire moves in that time. Now divide s by t. The result is the linear speed of the object.

Example 2 *A wheel in an amusement park has radius 20 feet and it makes a full rotation every 1.6 minutes. What is the linear speed of the wheel in (i) feet per second, (ii) yards per minute?*

***Solution**: If the wheel were to roll along the ground, it would move $20\,(2\pi)$ feet in 1.6 minutes. (We have chosen a time interval of 1.6 minutes since we know how far it rolls in that time.) Therefore its linear speed is*

$$\frac{40\pi}{1.6} \textit{ feet per minute.}$$

(i) Since there are 60 seconds in one minute, the linear speed in feet per second is

$$\left(\frac{40\pi}{1.6}\right) \div 60 = \frac{40\pi}{1.6 \times 60} \simeq 3.4$$

(ii) Since there are 3 feet in 1 yard, the linear speed in yards per minute is

$$\left(\frac{40\pi}{1.6}\right) \div 3 = \frac{40\pi}{1.6 \times 3} \simeq 26.2$$

Exercise 3

1. *Each tire of a truck has radius 1.8 feet. Assume that they make 10 complete revolutions per second. Calculate:*
   (a) *The angular speed of the tires in degrees per second.*
   (b) *The linear speed of the truck in feet per second.*
   (c) *The linear speed of the truck in miles per hour.*
2. *Each tire of a certain car has radius 1.2 feet. At what angular speed, in revolutions per second, are the tires turning when it is moving at a constant speed of 70 miles per hour? (The angular speed of the tires may be different from the angular speed of the engine because of the gears.)*

Angular Speed Formula

The angular speed of an object that rotates at a constant rate is given by the formula

$$\text{Angular Speed} \quad = \frac{\text{Angle swept out}}{\text{Time taken to sweep out the angle}}$$

A conventional symbol for angular speed is the greek letter ω, (pronounced "omega"). Therefore if a rotating object sweeps out an angle of x units, (these may be degrees or radians), in some specified time t, (this may be in seconds, minutes, hours, or other units of time), then its angular speed is

$$\omega = \frac{x}{t}.$$

Example 4 *Wanda noticed that when the ceiling fan in her room is set to operate at "very low" speed, it makes a full rotation in 5 seconds.*

This means that a point of the fan sweeps out an angle of 360° in 5 seconds. Therefore its angular speed is

$$\omega = \frac{360}{5} = 72 \text{ degrees per second}$$

If, instead, we measure angles in radians, then it sweeps out 2π radians in 5 seconds therefore its angular speed is

$$\omega = \frac{2\pi}{5} \text{ radians per second}$$

Linear Speed Formula

The linear speed of an object that is moving in a circle at a constant rate is the distance it travels per unit of time. To calculate it, one measures the distance s it travels along the circular path in a specified length of time t then divided s by t. A conventional symbol for speed is v. Therefore

$$v = \frac{s}{t}$$

Example 5 *Say a point at the end of a blade in the above fan is 26 inches from the center of rotation of the fan. Then in 5 seconds, such a point moves* $2\pi(26) = 52\pi$ *inches. Therefore its linear speed is*

$$\frac{52\pi}{5} \text{ inches per second.}$$

Rounded off to 1 decimal place, this is 32.7 *inches per second.*

Relation Between Angular Speed and Linear Speed

When a car is moving fast, its tires are rotating fast, and vice versa. Therefore angular speed and linear speed must be related by some equation. To determine such an equation, consider a car tire that is rotating. Say it has radius r feet and it rotates through x degrees in a period of t seconds. Then in those t seconds, the car travels a distance of

$$\frac{\pi r x}{180} \text{ feet}$$

Therefore its linear speed is

$$v = \frac{\pi r x}{180t} \text{ feet per second.}$$

Its angular speed is

$$\omega = \frac{x}{t} \text{ degrees per second}$$

Notice that we may write the linear speed as

$$v = \frac{\pi r}{180}\left(\frac{x}{t}\right) \text{ feet per second.}$$

If we replace $\left(\frac{x}{t}\right)$ by ω we conclude that the linear speed is

$$v = \frac{\pi r \omega}{180} \text{ feet per second.}$$

This is the required relation, if the angles are measured in degrees.

Exercise 6

1. *Give the relation between angular speed and linear speed when angles are measured in radians.*

2. *An object moves along a circle of radius 10 feet and it sweeps out an angle of* $60°$ *per second. Calculate its linear speed in (a) feet per second, (b) miles per hour.*

3. *An object moves along a circle of radius 30 feet and it sweeps out an angle of* $\frac{9\pi}{2}$ *radians per minute. Calculate its linear speed in (a) meters per minute, (b) kilometers per hour.*

4. *Find the linear speed in miles per hour of the tire with radius 14 inches and rotating at 1000 revolutions per minute.*

5. *Two objects A and B are moving along two circles with the same center and radii 5 feet and 20 feet respectively. If they move with the same angular speed, how are their linear speeds related?*

6. *Two objects A and B are moving along two circles with the same center and radii 10 feet and 15 feet respectively. If they move with the same linear speed, how are their angular speeds related?*